Period 3, Dec 18, 2024

$F'(x)$ is + $\Rightarrow$ $F(x)$ is increasing

$F'(x)$ is − $\Rightarrow$ $F(x)$ is decreasing

$F'(x) = 0 \Rightarrow F(x)$ is constant

$F''(x) = +$ concave up ☺ concave up

$F''(x) = -$ concave Down ☹ concave Down

and ter

Guidelines for Finding Intervals on which a Function is Increasing or Decreasing

1. Locate the critical number of f in (a, b) and use these numbers to determine test intervals.
2. Determine the sign (+ or -) of $f'(x)$ at one value in each of the intervals.
3. Use **Inc/Dec Test** to determine whether f is increasing or decreasing on each interval.

A function is strictly monotonic on an interval if it is either increasing on the entire interval or decreasing on the entire interval.

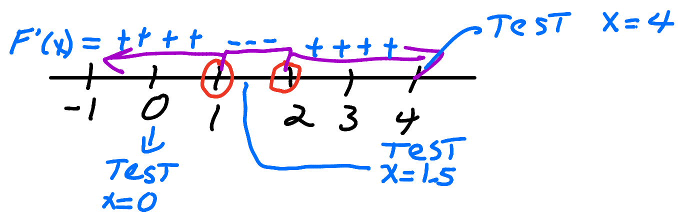
Example 7:

Determine where the function $f(x) = 2x^3 - 9x^2 + 12x - 5$
Is increasing and where it is decreasing.

Find $F'(x) = 6x^2 - 18x + 12 = 6(x^2 - 3x + 2) = 6(x-2)(x-1)$

critical #'s $x = 2, 1$

create intervals



Test values in intervals with $F'(x)$

$F'(0) = 6(0)^2 - 18(0) + 12$
$F'(0) = +12$

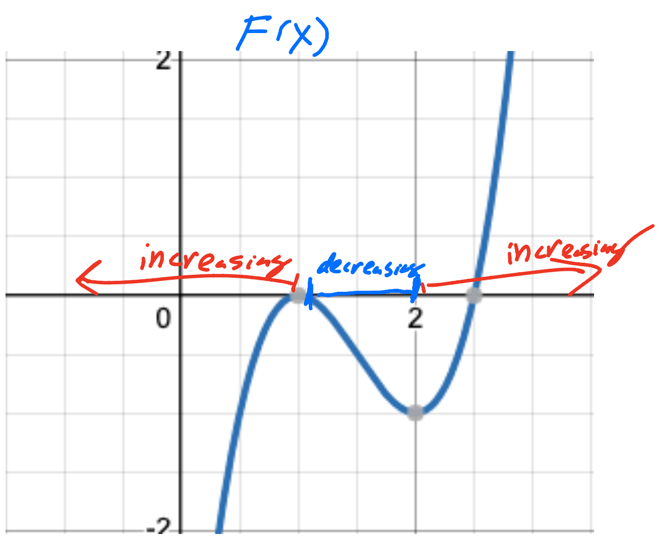
$F'(1.5) = 6(1.5-2)(1.5-1) = 6(-.5)(.5) = + \cdot - \cdot + = -$

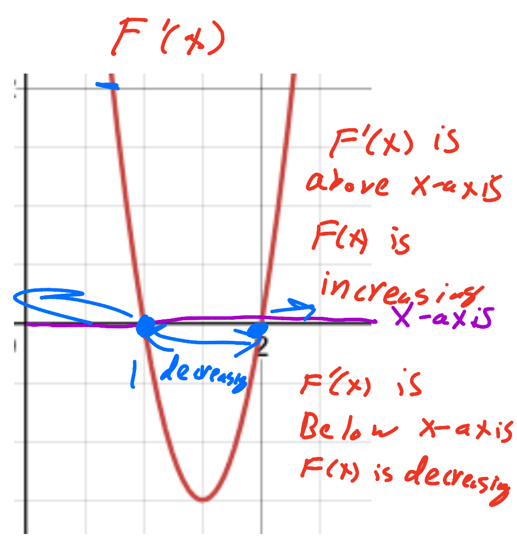
$F'(4) = 6(4-2)(4-1) = 6 \cdot 2 \cdot 3 = +$

determine intervals of increase and decrease of $F(x)$

increasing
$(-\infty, 1) \cup (2, \infty)$

decrease
$(1, 2)$





Determine where the function $f(x) = (x^2 - 1)^{2/3}$ is increasing and where it is decreasing.

$u = x^2 - 1$

$\frac{du}{dx} = 2x$

$y = (x^2-1)^{2/3}$

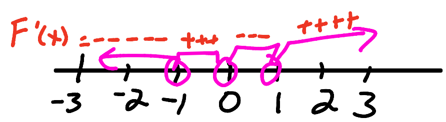
$y = u^{2/3}$

$\frac{dy}{du} = \frac{2}{3} u^{-\frac{1}{3}} = \frac{2}{3\sqrt[3]{u}}$

$\frac{du}{dx} \cdot \frac{dy}{du} = 2x \cdot \frac{2}{3\sqrt[3]{x^2-1}} = \frac{4x}{3\sqrt[3]{x^2-1}}$

$F'(x) = \frac{4x}{3\sqrt[3]{x^2-1}}$

critical #'s $x = 0, 1, -1$



increasing
$(-1,0) \cup (1,\infty)$

decreasing
$(-\infty,-1) \cup (0,1)$

Test

$F'(-2) = \frac{4(-2)}{3\sqrt[3]{(-2)^2-1}} = \frac{-}{+} = -$

$F'(-0.5) = \frac{4(-0.5)}{3\sqrt[3]{(-0.5)^2-1}} = \frac{-}{-} = +$

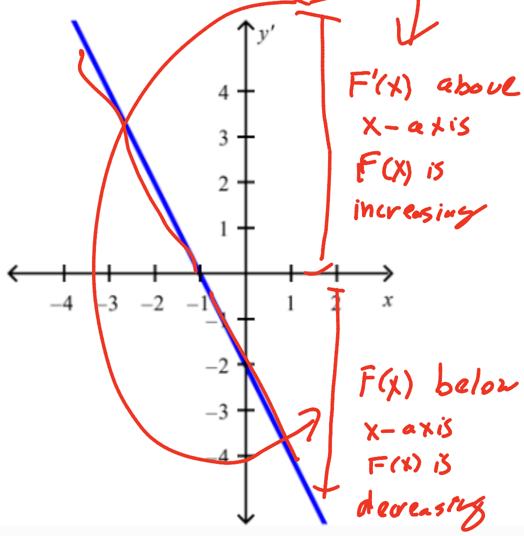
$- = F'(0.5) = \frac{4(0.5)}{3\sqrt[3]{(0.5)^2-1}} = \frac{+}{-}$

$F'(2) = \frac{4(2)}{3\sqrt[3]{2^2-1}} = \frac{+}{+} = +$

Identify intervals of increasing/decreasing when you are given f'.

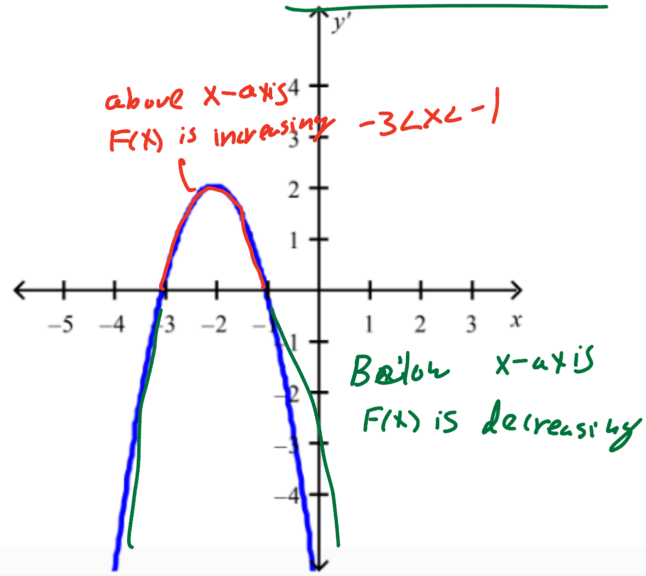
a) **Increasing** $(-\infty,-1)$

Decreasing $(-1,\infty)$



b) **Increasing** $(-3,-1)$

Decreasing $(-\infty,-3)U(-1,\infty)$



Example 9:

Suppose f is a function that is continuous for all real numbers. The graph of f' us shown.

a) What is the domain of the derivative f'? $(-\infty, 0) \cup (0, 2) \cup (2, \infty)$

b) List the critical numbers of f. $x = -2, 0, 2, 4$

c) On what intervals is the graph of f increasing? $(-2, 0) \cup (0, 2) \cup (2, 4)$

d) On what intervals is the graph of f decreasing? $(-\infty, -2) \cup (4, \infty)$

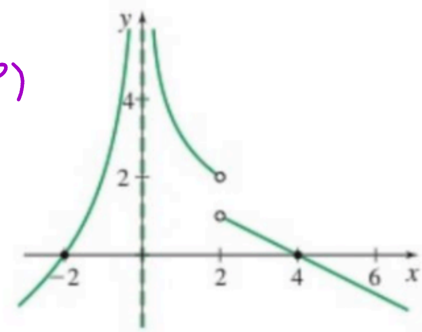


Figure 30 The graph of $y = f'(x)$

Example 9 (continued):

Suppose f is a function that is continuous for all real numbers. The graph of f' us shown.

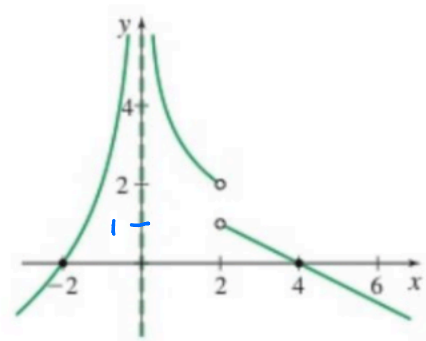
$F'(x)=0$

e) At what x values, if any, does the graph of f have a horizontal tangent $X=-2, X=4$

f) At what x values, if any, does the graph of f have a vertical tangent

$\lim_{x \to c} F'(x)=\infty$ $X=0$

g) At what x values, if any, does the graph of f have a corner

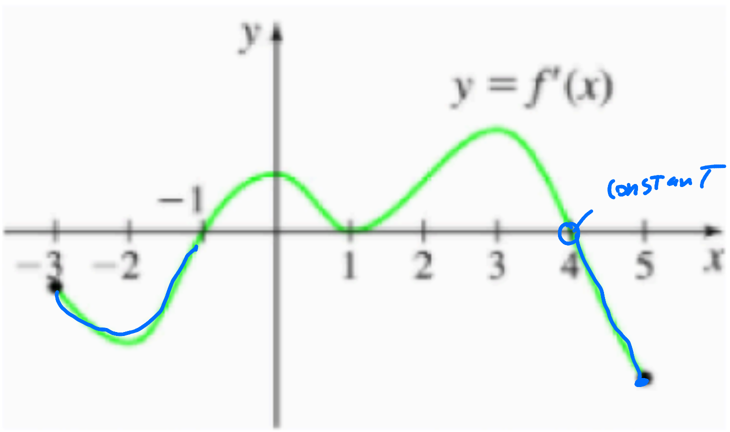


$\lim_{x \to 2^-} F'(x)=2$

$\lim_{x \to 2^+} F'(x)=1$

Sharp Turn (corner)

The graph of f' for the interval $[-3, 5]$ is shown below.



F'(x) is below The x-axis

On what interval(s) is (are) f decreasing?

$[-3, -1) \cup (4, 5]$

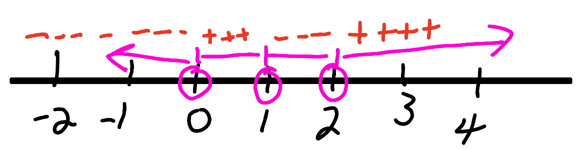
For which values of x is the function $f(x) = x^4 - 4x^3 + 4x^2 + 1$ decreasing?

$F'(x) = 4x^3 - 12x^2 + 8x = 4x(x^2 - 3x + 2)$

$F'(x) = 4x(x-1)(x-2)$

- $x < 0$ or $1 < x < 2$
- $x < 0$ or $x > 2$
- $0 < x < 2$
- $x > 2$ only

Critical #'s $x = 0, 1, 2$



$(-\infty, 0) \cup (1, 2)$

Test Points in intervals

decreasing

$F'(-2) = (4(-2))(-2-1)(-2-2) = - \cdot - \cdot - = -$

$F'(.5) = (4(0.5))(0.5-1)(0.5-2) = + \cdot - \cdot - = +$

$F'(1.5) = (4(1.5))(1.5-1)(1.5-2) = + \cdot + \cdot - = -$

$F'(3) = (4(3))(3-1)(3-2) = + \cdot + \cdot + = +$

An object moves along the x-axis so that its distance from the origin at any time $t \geq 0$ is given by $x(t) = t^3 + \frac{3}{2}t^2 - 18t + 4$.

At what times t is the object at rest? $V'(T)=0$

$V(T) = 3T^2 + 3T - 18 = 3(T^2 + T - 6) = 3(T+3)(T-2)$

$T = -3, 2$

$\cancel{V(-3)=0}$

$V(2)=0$

$\boxed{T=2}$

$\ln 3 > 1$ $\quad \ln e = 1$ $\quad e \approx 2.7$ $\quad \ln(1) = \emptyset$ $\quad \ln(1) = 0$

Which of the following statements is true for the function $f(x) = \dfrac{\ln x}{x}$, $x > 0$?

$F'(x) = \dfrac{\frac{1}{x}\cdot x - (\ln x)(1)}{x^2} = \dfrac{1 - \ln x}{x^2}$

Critical #'s $\quad X = 0$, $x = e$

$x = e$

+ + + + | − − − − −

−1 0 1 2 3 4 5

$\cancel{F'(-1) = \dfrac{1 - \ln(-1)}{(-1)^2} = \dfrac{1 - \ln(-1)}{1}}$

$+ = F'(1) = \dfrac{1 - \ln(1)}{(1)^2} = 1$

$- = F'(3) = \dfrac{1 - \ln(3)}{3^2} = \dfrac{-0.09}{9}$

$1 - 1.09$

- ◯ f is increasing on the interval $(0, \infty)$.
- ◯ f is increasing on the interval $[1, \infty)$.
- ◯ f is decreasing on the interval $[1, e]$.
- ◯ f is decreasing on the interval $[e, \infty)$